

Moving Mixed Branes in Compact Spacetime

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Abstract

In this article we present a general description of two moving branes in presence of the $B_{\mu\nu}$ field and gauge fields $A_{\alpha_1}^{(1)}$ and $A_{\alpha_2}^{(2)}$ on them, in spacetime in which some of its directions are compact on tori. Some examples are considered to elucidate this general description. Also contribution of the massless states to the interaction is extracted. Boundary state formalism is a useful tool for these considerations.

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1 Introduction

Boundary state formalism, which is a powerful tool for describing the branes and their interactions, has been successfully applied to a number of problems, for example D-branes dynamics in different configurations and spacetime dimensions [1, 2, 3, 4, 5, 6, 7]. On the other hand, back-ground fields $B_{\mu\nu}$ and A_α (a $U(1)$ gauge field which lives in brane) can be introduced to the string σ -model action, to obtain mixed boundary condition (i.e. a combination of Dirichlet and Neumann boundary conditions) for string [8]. Previously we obtained the mixed boundary state for a static mixed brane (i.e. a brane in above back-ground fields), and interaction of static mixed branes in spacetime in which some of its dimensions are compactified on tori [8]. We saw that the states emitted from the branes, which are wrapped around compact directions with internal back-ground fields turn out to be dominant along a certain direction. Their windings around the compact directions of brane are also correlated with their momenta along the brane.

In addition to the above considerations (i.e. existence of the back-ground fields and compactification of spacetime), now we consider the motion of the mixed branes. We will see that the momentum component of the closed string state along the motion of the brane, is also correlated with its windings around the compact directions of the brane. Also back-ground fields, compactification and velocities all together, cause the interaction amplitude take an interesting form. For example when these three exist, the initial position y^{i_0} of the brane along the motion appears in the interaction.

In section 2 we obtain the boundary states for moving mixed branes in compact spacetime. In section 3 we use of these boundary states to calculate interaction of two branes of dimensions p_1 and p_2 with different internal fields \mathcal{F}_1 and \mathcal{F}_2 , moving with velocities V_1 and V_2 . We shall also show that these results reduce to the known cases of the D-branes in non-compact spacetime. To elucidate our general computations, we apply our results to special cases: parallel $m_1 - m_{1'}$ and perpendicular $m_1 - m_{1'}$ systems. Finally contribution of the massless states on the interaction will be obtained.

In this article a brane in back-ground internal fields, is denoted by “ m_p -brane”, which is a “mixed brane” with dimension “ p ”. Since compactification effects on the interaction of the moving mixed branes do not depend on the fermions, we will consider only the bosonic string.

2 Moving mixed brane and boundary state

We begin with a σ -model action containing $B_{\mu\nu}$ field and two boundary terms corresponding to the two m_{p_1} and m_{p_2} -branes gauge fields [9], and their velocities [4, 10]

$$\begin{aligned}
S = & -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma \left(\sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right) \\
& -\frac{1}{2\pi\alpha'} \int_{(\partial\Sigma)_1} d\sigma \left(A_{\alpha_1}^{(1)} \partial_\sigma X^{\alpha_1} + V_1^{i_1} X^0 \partial_\tau X^{i_1} \right) \\
& +\frac{1}{2\pi\alpha'} \int_{(\partial\Sigma)_2} d\sigma \left(A_{\alpha_2}^{(2)} \partial_\sigma X^{\alpha_2} + V_2^{i_2} X^0 \partial_\tau X^{i_2} \right) , \tag{1}
\end{aligned}$$

where Σ is the world sheet of the closed string exchanged between the branes. $(\partial\Sigma)_1$ and $(\partial\Sigma)_2$ are two boundaries of this world sheet, which are at $\tau = 0$ and $\tau = \tau_0$ respectively. $A_{\alpha_1}^{(1)}$ and $A_{\alpha_2}^{(2)}$ are $U(1)$ gauge fields that live in m_{p_1} and m_{p_2} -branes. $V_1^{i_1}$ and $V_2^{i_2}$ are velocities of the first and the second branes. The sets $\{\alpha_1\}$ and $\{\alpha_2\}$ specify the directions on the m_{p_1} and m_{p_2} world volumes, $\{i_1\}$ and $\{i_2\}$ show the directions perpendicular to them.

Taking the back-ground fields $G_{\mu\nu}(X)$ and $B_{\mu\nu}(X)$ to be constant fields. Vanishing the variation of this action with respect to $X^\mu(\sigma, \tau)$ gives the equation of motion of $X^\mu(\sigma, \tau)$ and boundary state equations. For the second brane, boundary state equations take the form,

$$\left(\partial_\tau (X^0 - V_2^{i_2} X^{i_2}) + \mathcal{F}_{(2)\beta_2}^0 \partial_\sigma X^{\beta_2} - B^0{}_{i_2} \partial_\sigma (X^{i_2} - V_2^{i_2} X^0) \right)_{\tau=\tau_0} |B_x^2, \tau_0\rangle = 0 , \tag{2}$$

$$\left(\partial_\tau X^{\bar{\alpha}_2} + \mathcal{F}_{(2)\beta_2}^{\bar{\alpha}_2} \partial_\sigma X^{\beta_2} - B^{\bar{\alpha}_2}{}_{i_2} \partial_\sigma (X^{i_2} - V_2^{i_2} X^0) \right)_{\tau=\tau_0} |B_x^2, \tau_0\rangle = 0 , \tag{3}$$

$$\delta(X^{i_2} - V_2^{i_2} X^0)_{\tau=\tau_0} |B_x^2, \tau_0\rangle = 0 , \tag{4}$$

where $\bar{\alpha}_2$ refers to the spatial directions of the m_{p_2} -brane (i.e. $\bar{\alpha}_2 \neq 0$), and \mathcal{F}_2 is total “field strength”,

$$\mathcal{F}_{(2)\alpha_2\beta_2} \equiv \partial_{\alpha_2} A_{\beta_2}^{(2)} - \partial_{\beta_2} A_{\alpha_2}^{(2)} - B_{\alpha_2\beta_2} . \tag{5}$$

The transverse coordinates of the two branes initially are $\{y_1^{i_1}\}$ and $\{y_2^{i_2}\}$, therefore

$$\left(X^{i_2}(\sigma, \tau) - V_2^{i_2} X^0(\sigma, \tau) - y_2^{i_2} \right)_{\tau=\tau_0} |B_x^2, \tau_0\rangle = 0 . \tag{6}$$

This implies $\partial_\sigma (X^{i_2} - V_2^{i_2} X^0)$ vanish on the boundary and be dropped from the equations (2) and (3).

Solution of the equation of motion of the closed string is

$$X^\mu(\sigma, \tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2} \sqrt{2\alpha'} \sum_{m \neq 0} \frac{1}{m} (\alpha_m^\mu e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m^\mu e^{-2im(\tau+\sigma)}) , \quad (7)$$

where L^μ is zero for non-compact directions, for compact directions we have $L^\mu = N^\mu R^\mu$ and $p^\mu = \frac{M^\mu}{R^\mu}$, in which N^μ is the winding number and M^μ is the momentum number of the closed string state, also R_μ is the radius of compactification in the compact direction X^μ .

Combining the solution of the equation of motion and the boundary state equations, assuming non-compact time direction, we obtain the boundary state equations in terms of modes,

$$\left[\left(\alpha_m^0 - V_2^{i_2} \alpha_m^{i_2} - \mathcal{F}_{(2)}^0{}_{\bar{\beta}_2} \alpha_m^{\bar{\beta}_2} \right) e^{-2im\tau_0} + \left(\tilde{\alpha}_{-m}^0 - V_2^{i_2} \tilde{\alpha}_{-m}^{i_2} + \mathcal{F}_{(2)}^0{}_{\bar{\beta}_2} \tilde{\alpha}_{-m}^{\bar{\beta}_2} \right) e^{2im\tau_0} \right] |B_x^2, \tau_0\rangle = 0 , \quad (8)$$

$$\left[\left(\alpha_m^{\bar{\alpha}_2} - \mathcal{F}_{(2)}^{\bar{\alpha}_2}{}_{\beta_2} \alpha_m^{\beta_2} \right) e^{-2im\tau_0} + \left(\tilde{\alpha}_{-m}^{\bar{\alpha}_2} + \mathcal{F}_{(2)}^{\bar{\alpha}_2}{}_{\beta_2} \tilde{\alpha}_{-m}^{\beta_2} \right) e^{2im\tau_0} \right] |B_x^2, \tau_0\rangle = 0 , \quad (9)$$

$$\left[\left(\alpha_m^{i_2} - V_2^{i_2} \alpha_m^0 \right) e^{-2im\tau_0} - \left(\tilde{\alpha}_{-m}^{i_2} - V_2^{i_2} \tilde{\alpha}_{-m}^0 \right) e^{2im\tau_0} \right] |B_x^2, \tau_0\rangle = 0 , \quad (10)$$

for the oscillating part, and

$$\left(p^0 - V_2^{i_2} p^{i_2} + \frac{1}{\alpha'} \mathcal{F}_{(2)}^0{}_{\bar{\beta}_2} L^{\bar{\beta}_2} \right)_{op} |B_x^2, \tau_0\rangle = 0 , \quad (11)$$

$$\left(p^{\bar{\alpha}_2} + \frac{1}{\alpha'} \mathcal{F}_{(2)}^{\bar{\alpha}_2}{}_{\bar{\beta}_2} L^{\bar{\beta}_2} \right)_{op} |B_x^2, \tau_0\rangle = 0 , \quad (12)$$

$$\left(x^{i_2} - V_2^{i_2} x^0 - y_2^{i_2} + 2\alpha' \tau_0 (p^{i_2} - V_2^{i_2} p^0) \right)_{op} |B_x^2, \tau_0\rangle = 0 , \quad (13)$$

$$L^{i_2} |B_x^2, \tau_0\rangle = 0 , \quad (14)$$

for the zero mode part. The oscillating part can be written as,

$$\left(\alpha_m^\mu e^{-2im\tau_0} + S^\mu{}_\nu \tilde{\alpha}_{-m}^\nu e^{2im\tau_0} \right) |B_x^2, \tau_0\rangle = 0 , \quad (15)$$

$$S \equiv M^{-1} N , \quad (16)$$

where matrices M and N , which depend on \mathcal{F}_2 and V_2 are defined by

$$\begin{cases} M^0_{\mu} = \delta^0_{\mu} - V_2^{i_2} \delta^{i_2}_{\mu} - \mathcal{F}_{(2)\beta_2}^0 \delta^{\beta_2}_{\mu} \\ M^{\bar{\alpha}_2}_{\mu} = \delta^{\bar{\alpha}_2}_{\mu} - \mathcal{F}_{(2)\beta_2}^{\bar{\alpha}_2} \delta^{\beta_2}_{\mu} \\ M^{i_2}_{\mu} = \delta^{i_2}_{\mu} - V_2^{i_2} \delta^0_{\mu} \end{cases} \quad (17)$$

and

$$\begin{cases} N^0_{\mu} = \delta^0_{\mu} - V_2^{i_2} \delta^{i_2}_{\mu} + \mathcal{F}_{(2)\beta_2}^0 \delta^{\beta_2}_{\mu} \\ N^{\bar{\alpha}_2}_{\mu} = \delta^{\bar{\alpha}_2}_{\mu} + \mathcal{F}_{(2)\beta_2}^{\bar{\alpha}_2} \delta^{\beta_2}_{\mu} \\ N^{i_2}_{\mu} = -\delta^{i_2}_{\mu} + V_2^{i_2} \delta^0_{\mu} \end{cases} \quad (18)$$

These definitions of the matrices M and N imply S be an orthogonal matrix, i.e. $MM^T = NN^T$, one can investigate this identity element by element.

We now extract the boundary state from the equations (11-15). Oscillators in (15) results in

$$|B_{osc}^2\rangle = \exp\left[-\sum_{m=1}^{\infty} \left(\frac{1}{m} e^{4im\tau_0} \alpha_{-m}^{\mu} S_{\mu\nu}^{(2)} \tilde{\alpha}_{-m}^{\nu}\right)\right] |0\rangle . \quad (19)$$

From now on, we restrict ourselves to the case that m_{p_1} and m_{p_2} -branes move along the x^{i_0} -direction which is perpendicular to the both of them, therefore $V_1^{i_0} \equiv V_1$ and $V_2^{i_0} \equiv V_2$ and all other components of the velocities are zero. These imply the solutions of the zero mode part to be as the following,

$$|B_x^2, \tau_0\rangle^{(0)} = \sum_{\{p^{\alpha_2}\}} |B_x^2, \tau_0, p^{\alpha_2}\rangle^{(0)} , \quad (20)$$

$$\begin{aligned} |B_x^2, \tau_0, p^{\alpha_2}\rangle^{(0)} &= \frac{T_{p_2}}{2} \sqrt{\det M_2} \exp\left[i\alpha' \tau_0 \left(\gamma_2^2 (p_{op}^{i_0} - V_2 p_{op}^0)^2 + \sum_{j_2 \neq i_0} (p_{op}^{j_2})^2\right)\right] \\ &\times \delta(x^{i_0} - V_2 x^0 - y_2^{i_0}) \prod_{j_2 \neq i_0} \delta(x^{j_2} - y_2^{j_2}) \prod_{\bar{\alpha}_2} |p^{\bar{\alpha}_2}\rangle \\ &\times \prod_{j_2 \neq i_0} |p_L^{j_2} = p_R^{j_2} = 0\rangle |p^0\rangle |p_L^{i_0} = p_R^{i_0} = \frac{1}{2} V_2 p^0\rangle \end{aligned} \quad (21)$$

where γ_2 is $1/\sqrt{1-V_2^2}$ and T_{p_2} is the D_{p_2} -brane tension [11]. Path integral with boundary action gives $\sqrt{\det M_2}$ [12, 13, 14], and for the $\mathcal{F}_2 = 0$ it becomes $\frac{1}{\gamma_2}$. In this state momentum components are,

$$p^{\bar{\alpha}_2} = -\frac{1}{\alpha'} \mathcal{F}_{(2)\beta_{2c}}^{\bar{\alpha}_2} \ell^{\beta_{2c}} , \quad (22)$$

$$p^0 = -\frac{\gamma_2^2}{\alpha'} \mathcal{F}_{(2)}^0 \ell^{\beta_{2c}} , \quad (23)$$

$$p^{i_0} = -\frac{V_2 \gamma_2^2}{\alpha'} \mathcal{F}_{(2)}^0 \ell^{\beta_{2c}} , \quad (24)$$

therefore, for the closed string state emitted from the moving brane with back-ground fields in compact spacetime, besides, that the momentum components along the world volume of the brane are non-zero and are quantized, the momentum component along the motion of the brane is also non-zero and is quantized. More details of (22-24) for $V_2 = 0$, can be found in [8]. In (20), due to the relations (22-24), the summation over the momentum components can be changed to a sum over winding numbers, $\{N^{\alpha_{2c}}\}$.

Ghost part of the boundary state has the form

$$|B_{gh}, \tau_0\rangle = \exp\left[\sum_{m=1}^{\infty} e^{4im\tau_0} (c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m})\right] \frac{c_0 + \tilde{c}_0}{2} |q=1\rangle |\tilde{q}=1\rangle \quad (25)$$

3 Moving mixed branes interaction

Before calculation of the interaction amplitude, let us introduce some notations for the positions of these two mixed branes. The set $\{\bar{i}\}$ shows the directions perpendicular to the both of the branes, in which i_0 is not in $\{\bar{i}\}$, the set $\{\bar{u}\}$ for the directions along the both of them, in which 0 is not in $\{\bar{u}\}$, the set $\{\alpha'_1\}$ for the directions along the m_{p_1} and perpendicular to the m_{p_2} , and the set $\{\alpha'_2\}$ for the directions along the m_{p_2} and perpendicular to the m_{p_1} -branes. It can be seen that for example

$$\{i_1\} = \{\bar{i}\} \cup \{i_0\} \cup \{\alpha'_2\} , \quad \{\alpha_1\} = \{\alpha'_1\} \cup \{\bar{u}\} \cup \{0\} . \quad (26)$$

The complete boundary state can be written as the product $|B\rangle = |B_x\rangle |B_{gh}\rangle$, therefore the interaction amplitude is

$$\mathcal{A} = \langle B_1 | D | B_2, \tau_0 = 0 \rangle , \quad (27)$$

where “ D ” is the closed string propagator. The final result is

$$\begin{aligned} \mathcal{A} = & \frac{T_{p_1} T_{p_2}}{4(2\pi)^{d_i}} \frac{\alpha' \gamma_1 \gamma_2}{\sinh \omega} \sqrt{\det M_1 \det M_2} \int_0^\infty dt \left\{ e^{4at} \right. \\ & \times \prod_{n=1}^{\infty} \left([\det(1 - S_1 S_2^T e^{-4nt})]^{-1} (1 - e^{-4nt})^2 \right) \end{aligned}$$

$$\begin{aligned}
& \times \left(\sqrt{\frac{\pi}{\alpha't}} \right)^{\bar{d}_{i_n}} e^{-\frac{1}{4\alpha't} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2} \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha't}{\pi(R_{\bar{i}_c})^2} \right) \\
& \times \sum_{\{N^{u_c}\}} \left[(2\pi)^{\bar{d}_u} \left[\prod_{\bar{u}} \delta(p_1^{\bar{u}} - p_2^{\bar{u}}) \right] \exp \left[\frac{i}{\alpha'} \ell^{u_c} \left(\mathcal{F}_{(1) \ u_c}^{\alpha'_1} y_2^{\alpha'_1} - \mathcal{F}_{(2) \ u_c}^{\alpha'_2} y_1^{\alpha'_2} \right. \right. \right. \\
& \left. \left. \left. + \phi_{u_c}(12) y_2^{i_0} - \phi_{u_c}(21) y_1^{i_0} \right) \right] \exp \left[-\frac{t}{\alpha'} \ell^{u_c} \ell^{v_c} \left(\delta_{u_c v_c} + f_{u_c}^{(+)} f_{v_c}^{(-)} \right. \right. \right. \\
& \left. \left. \left. + \mathcal{F}_{(1) \ u_c}^{\bar{u}} \mathcal{F}_{(2) \ \bar{u} v_c} + \mathcal{F}_{(1) \ u_c}^{\alpha'_1} \mathcal{F}_{(1) \ v_c}^{\alpha'_1} + \mathcal{F}_{(2) \ u_c}^{\alpha'_2} \mathcal{F}_{(2) \ v_c}^{\alpha'_2} \right) \right] \right] \Big\} \quad (28)
\end{aligned}$$

where a , ω , $\phi_{u_c}(12)$, and $f_{u_c}^{(+)}$ are

$$a = (d-2)/24 \quad , \quad \omega = |\omega_2 - \omega_1| \quad , \quad V_{1,2} = tgh\omega_{1,2} \quad , \quad (29)$$

$$\phi_{u_c}(12) = \frac{1}{V_2 - V_1} \left[\gamma_2^2 (1 + V_1 V_2) \mathcal{F}_{(2) \ u_c}^0 - \gamma_1^2 (1 + V_1^2) \mathcal{F}_{(1) \ u_c}^0 \right] \quad , \quad (30)$$

$$f_{u_c}^{(+)} = \frac{1}{|V_1 - V_2|} \left[\gamma_2^2 (1 + V_1)(1 + V_2^2) \mathcal{F}_{(2) \ u_c}^0 - \gamma_1^2 (1 + V_2)(1 + V_1^2) \mathcal{F}_{(1) \ u_c}^0 \right] \quad , \quad (31)$$

and $\phi_{u_c}(21)$ is given in (31) with the exchange $1 \leftrightarrow 2$, also for $f_{u_c}^{(-)}$, change the signs of V_1 and V_2 in (32).

In this amplitude $p_1^{\bar{u}} = -\frac{1}{\alpha'} \mathcal{F}_{(1) \ v_c}^{\bar{u}} N^{v_c} R^{v_c}$ and $p_2^{\bar{u}} = -\frac{1}{\alpha'} \mathcal{F}_{(2) \ v_c}^{\bar{u}} N^{v_c} R^{v_c}$. Indices $\{u_c, v_c\}$ show the compact part of $\{\bar{u}\}$, also \bar{d}_i and \bar{d}_{i_n} show the dimensions of $\{X^{\bar{i}}\}$ and $\{X^{\bar{i}_n}\}$ respectively. The sets $\{\bar{i}_n\}$ and $\{\bar{i}_c\}$ show the non-compact and compact part of $\{\bar{i}\}$. Under the exchange of indices “1” and “2” this amplitude is symmetric i.e. $\mathcal{A}_{(1,2)} = \mathcal{A}_{(2,1)}^*$, as expected (see(28)). From (29) and (31) we see that the non-zero electric fields, $E_{u_c}^{(1)} = \mathcal{F}_{(1) \ 0u_c}^0$ and $E_{u_c}^{(2)} = \mathcal{F}_{(2) \ 0u_c}^0$, spacetime compactification and motion of the branes cause the $y_1^{i_0}$ and $y_2^{i_0}$ to appear in the interaction.

The momentum delta functions put some restrictions on the summation. The term corresponding to $N^{u_c} = 0$ for all u_c , gives $p_1^{\bar{u}} = p_2^{\bar{u}} = 0$, and is always present. Other terms occur only if the two internal back-ground fields and radii of compactification with some sets $\{N_s^{u_c}\}$ satisfy the relation $\sum_{v_c} (\mathcal{F}_{(1) \ v_c}^{\bar{u}} N_s^{v_c} R^{v_c}) = \sum_{v_c} (\mathcal{F}_{(2) \ v_c}^{\bar{u}} N_s^{v_c} R^{v_c})$ for all \bar{u} . In this case common volume of the branes ($V_{\bar{u}}$) explicitly appears in the amplitude, therefore

$$\begin{aligned}
\mathcal{A} &= \frac{T_{p_1} T_{p_2} V_{\bar{u}}}{4(2\pi)^{\bar{d}_i}} \frac{\alpha' \gamma_1 \gamma_2}{\sinh \omega} \sqrt{\det M_1 \det M_2} \int_0^\infty dt \left\{ e^{4at} \right. \\
& \times \prod_{n=1}^\infty \left([\det(1 - S_1 S_2^T e^{-4nt})]^{-1} (1 - e^{-4nt})^2 \right) \\
& \times \left(\sqrt{\frac{\pi}{\alpha't}} \right)^{\bar{d}_{i_n}} e^{-\frac{1}{4\alpha't} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2} \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha't}{\pi(R_{\bar{i}_c})^2} \right)
\end{aligned}$$

$$\times \left[\left[1 + \sum_s \{ \exp \left[-\frac{t}{\alpha'} \ell_s^{u_c} \ell_s^{v_c} \left(\delta_{u_c v_c} + f_{u_c}^{(+)} f_{v_c}^{(-)} + \mathcal{F}_{(1) u_c}^{\bar{u}} \mathcal{F}_{(2) \bar{u} v_c} + \mathcal{F}_{(1) u_c}^{\alpha'_1} \mathcal{F}_{(1) v_c}^{\alpha'_1} + \mathcal{F}_{(2) u_c}^{\alpha'_2} \mathcal{F}_{(2) v_c}^{\alpha'_2} \right) \right] \exp \left[\frac{i}{\alpha'} \ell_s^{u_c} \left(\mathcal{F}_{(1) u_c}^{\alpha'_1} y_2^{\alpha'_1} - \mathcal{F}_{(2) u_c}^{\alpha'_2} y_1^{\alpha'_2} + \phi_{u_c}(12) y_2^{i_0} - \phi_{u_c}(21) y_1^{i_0} \right) \right] \} \right] \right] \} \} \quad (32)$$

where $\ell_s^{u_c} = N_s^{u_c} R^{u_c}$. If there are no sets $\{N_s^{u_c}\}$ then $[[\quad]] = 1$. For parallel mixed branes with the same dimension those terms containing α'_1 and α'_2 disappear.

The all effects of compactification are in the last bracket and the products of Θ_3 -functions, therefore amplitude in non-compact spacetime is

$$\begin{aligned} \mathcal{A}_{(nc)} &= \frac{T_{p_1} T_{p_2} V_{\bar{u}}}{4(2\pi)^{\bar{d}_i}} \frac{\alpha' \gamma_1 \gamma_2}{\sinh \omega} \sqrt{\det M_1 \det M_2} \int_0^\infty dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{\bar{d}_i} e^{-\frac{1}{4\alpha' t} \sum_i (y_1^{\bar{i}} - y_2^{\bar{i}})^2} \right. \\ &\quad \times e^{4at} \prod_{n=1}^\infty \left([\det(1 - S_1 S_2^T e^{-4nt})]^{-1} (1 - e^{-4nt})^2 \right) \left. \right\}. \end{aligned} \quad (33)$$

For parallel D_p -branes (i.e. $\mathcal{F}_1 = \mathcal{F}_2 = 0$) along (X^1, X^2, \dots, X^p) with $T_p = \frac{\sqrt{\pi}}{2(d-10)/4} (4\pi^2 \alpha')^{(d-2p-4)/4}$ and $t \rightarrow \pi t/2$, the amplitude $\mathcal{A}_{(nc)}$ reduces to result of [5].

4 Examples

To elucidate these general computations, we apply our results to special cases. These are: parallel $m_1 - m_{1'}$ branes along X^1 -direction, moving along X^2 -direction and perpendicular $m_1 - m_{1'}$ branes along X^1 and X^2 directions, moving along X^3 -direction. For both of these examples we give the following amplitude,

$$\begin{aligned} \mathcal{A}_{(1,1')} &= \frac{T_1^2 L}{4(2\pi)^{d-r}} \frac{\alpha' \gamma_1 \gamma_2}{\sinh \omega} \sqrt{(1 - E_1^2 - V_1^2)(1 - E_2^2 - V_2^2)} \int_0^\infty dt \left\{ \right. \\ &\quad \times e^{4at} \prod_{n=1}^\infty \left([\det(1 - H_1 H_2^T e^{-4nt})]^{-1} (1 - e^{-4nt})^{2+r-d} \right) \\ &\quad \times \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{\bar{d}_{in}} e^{-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2} \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2} \right) \\ &\quad \times \theta(t, R, V, \mathcal{F}) \left. \right\}. \end{aligned} \quad (34)$$

parallel m_1 -branes

For this system we have $L = 2\pi R_1$, $r = 3$, $E_1 = \mathcal{F}_{(1) 01}$, $E_2 = \mathcal{F}_{(2) 01}$, $\bar{i} \in \{3, 4, \dots, d-1\}$ and function θ is

$$\theta(E_1, E_2, V_1, V_2, t, R_1) = \Theta_3 \left(\frac{(\phi_{12} y_2^2 - \phi_{21} y_1^2) R_1}{2\pi \alpha'} \mid \frac{it(1 + f_+ f_-) R_1^2}{\pi \alpha'} \right), \quad (35)$$

where ϕ_{12} and f_+ are the same as in (31) and (32) with $u_c = 1$, and corresponding expressions for ϕ_{21} and f_- . If X^1 -direction is not compact then $\theta = 1$, and therefore interaction will be

independent to y_1^2, y_2^2, ϕ_{12} and f_{\pm} . Also matrices H_1 and H_2 have similar form, for simplicity drop the indices 1 and 2, therefore

$$H^p_q = \frac{1}{1 - E^2 - V^2} \begin{pmatrix} 1 + E^2 + V^2 & -2E & -2V \\ -2E & 1 + E^2 - V^2 & 2EV \\ 2V & -2EV & -(1 - E^2 + V^2) \end{pmatrix}, \quad (36)$$

where $p, q = 0, 1, 2$. Note that for this system matrices S_1 and S_2 have the common form

$$S^\mu_\nu = \begin{pmatrix} H^p_q & 0 \\ 0 & -\mathbf{1}_{(d-3) \times (d-3)} \end{pmatrix}. \quad (37)$$

In this example the matrix $S_{\mu\nu} = \eta_{\mu\lambda} S^\lambda_\nu$ is exactly that, which is given in Ref.[4], with notation $(\eta.\Lambda.T)_{\mu\nu}$, (our definition of E is negative of [4]).

perpendicular \mathbf{m}_1 -branes

For this system there are $L = 1, r = 4, E_1 = \mathcal{F}_{(1)01}, E_2 = \mathcal{F}_{(2)02}, \bar{i} \in \{4, 5, \dots, d - 1\}, \theta(t, R, V, \mathcal{F}) = 1$ and matrices H_1 and H_2 are

$$H_2 = \frac{1}{1 - E_2^2 - V_2^2} \begin{pmatrix} 1 + E_2^2 + V_2^2 & 0 & -2E_2 & -2V_2 \\ 0 & -(1 - E_2^2 - V_2^2) & 0 & 0 \\ -2E_2 & 0 & 1 + E_2^2 - V_2^2 & 2E_2V_2 \\ 2V_2 & 0 & -2E_2V_2 & -(1 - E_2^2 + V_2^2) \end{pmatrix}, \quad (38)$$

and matrix H_1 can be obtained from the H_2 as the following: change the second and third columns with each other and again in this new matrix change the second and third rows with each other, finally change the index “2” to “1”.

5 Massless states contribution to the amplitude

In this part, to see how distant branes interact we obtain the interaction of these branes due to the exchange of the massless states. As the metric, antisymmetric tensor and dilaton states have zero winding and zero momentum numbers, only the term with $N^{u_c} = 0$ (for all u_c) corresponds to these three massless states. By using the identity $\det A = e^{Tr(\ln A)}$ for a matrix A , there is the following limit for $d = 26$,

$$\lim_{q \rightarrow 0} \frac{1}{q} \prod_{n=1}^{\infty} \left[(1 - q^n)^2 [\det(1 - S_1 S_2^T q^n)]^{-1} \right] = \lim_{q \rightarrow 0} \frac{1}{q} + \left(Tr(S_1 S_2^T) - 2 \right), \quad (39)$$

where $q = e^{-4t}$, putting out the tachyon divergence, contribution of these three massless states becomes

$$\begin{aligned} \mathcal{A}_0 = & \frac{T_{p_1} T_{p_2} V_{\bar{u}}}{4(2\pi)^{d_i}} \frac{\alpha' \gamma_1 \gamma_2}{\sinh \omega} \sqrt{\det M_1 \det M_2} [Tr(S_1 S_2^T) - 2] \int_0^\infty dt \left[\left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{\bar{d}_{in}} \right. \\ & \left. \times e^{-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2} \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2} \right) \right] . \end{aligned} \quad (40)$$

We see that integrand completely comes from the directions perpendicular to the both of the branes (except the direction of motion X^{i_0}), it also is independent of the fields and velocities of the branes. For the parallel m_p -branes in non-compact spacetime, this reads as

$$\mathcal{A}_0 = \frac{T_p^2}{4} \frac{\gamma_1 \gamma_2 V_p}{\sinh \omega} \sqrt{\det M_1 \det M_2} [Tr(S_1 S_2^T) - 2] G_{24-p}(\bar{Y}^2) , \quad (41)$$

where $\bar{Y}^2 = \sum_{\bar{i}} (y_1^{\bar{i}} - y_2^{\bar{i}})^2$ is impact parameter.

6 Conclusion

We explicitly showed that how total field strength, velocity of the brane and compactification effects appear in the boundary state. These cause the closed string state emitted from the brane to have a quantized momentum along the brane and, along the motion of the brane.

Interaction amplitude takes the general form under the influence of total field strengths $(\mathcal{F}_1, \mathcal{F}_2)$, velocities (V_1, V_2) , dimensions (p_1, p_2) and compactification. In non-compact spacetime exchange of the massless states between the parallel m_p -branes depends on the impact parameter as $1/|\bar{Y}|^{(22-p)}$.

The formalism can be extended to include type IIA and type IIB superstring theories.

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